Λύσεις Διαγωνίσματος 9/5/2021 Επαναληπτικό.

ΘΕΜΑ Α

$A_1-\beta$, $A_2-\alpha$, $A_3-\beta$, $A_4-\gamma$ A_5 ΣΛΣΣ

ΘΕΜΑ Β

Β1 Ι-β $A_{ολ} = \sqrt{A_1^2+A_2^2} = \sqrt{2A^2+A^2} = A\sqrt{3}$

$E = \frac{1}{2}DA_{ολ}^2 = \frac{1}{2}m\omega^2 3A^2 \Rightarrow \boxed{E = \frac{3}{2}m\omega^2A^2}$

ΙΙ-α $\varepsilon\varphi\varphi_0 = \frac{A_1}{A_2} = \sqrt{2}$ $\varphi_0 = \frac{\pi}{4}$ Αρχική επαλληλίας $x = x_1+x_2$

Την $t = T/4$ $x = A\sqrt{2}\eta\mu\left(\frac{2\pi}{T}\frac{T}{4}+\frac{\pi}{2}\right) + A\eta\mu\left(\frac{2\pi}{T}\frac{T}{4}\right)$

$\Rightarrow x = A\sqrt{2}\eta\mu(\pi) + A\eta\mu\frac{\pi}{2} \Rightarrow \boxed{x = +A}$ ($\eta\mu\pi = 0$, $\eta\mu\frac{\pi}{2} = 1$)

Β2-α $K_1' = 25\%\, K_1 \Rightarrow \frac{1}{2}m_1v_1'^2 = \frac{1}{4}\frac{1}{2}m_1v_1^2 \Rightarrow |v_1'| = \frac{v_1}{2}$

$v_1' = \frac{m_1-m_2}{m_1+m_2}v_1 \Rightarrow -\frac{v_1}{2} = \frac{m_1-m_2}{m_1+m_2}v_1 \Rightarrow -\frac{1}{2} = \frac{m_1-m_2}{m_1+m_2}$

$\Rightarrow -m_1-m_2 = 2m_1-2m_2 \Rightarrow m_2 = 3m_1 \Rightarrow 3m = 3m_1 \Rightarrow m_1 = m$

ΑΔΟ για πλαστική κρούση: $\vec{P}_{πριν} = \vec{P}_{μετα} \Rightarrow m_1|v_1'| = (m_1+m_3)v_K$

$\Rightarrow m\frac{v_1}{2} = 2mv_K \Rightarrow v_K = \frac{v_1}{4}$

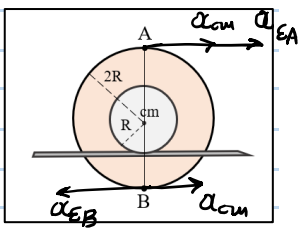
$K_{πριν} = K_1' = \frac{1}{2}m_1v_1'^2 = \frac{1}{2}m\frac{v_1^2}{4} = \frac{1}{4}\frac{1}{2}mv_1^2 = \frac{1}{4}K_1$

$K_{μετα} = K_{κοινη} = \frac{1}{2}(m_1+m_3)v_K^2 = \frac{1}{2}2m\frac{v_1^2}{16} = \frac{1}{8}\frac{1}{2}mv_1^2 = \frac{1}{8}K_1$

$E_{απωλ} = K_{πριν} - K_{μετα} = \frac{1}{4}K_1 - \frac{1}{8}K_1 \Rightarrow \boxed{E_{απωλ} = \frac{1}{8}K_1}$

Β3 Ι-γ κ×ο $a_{cm} = R\alpha_{γων}$ $a_{εΑ} = 2R\alpha_{γων} = 2a_{cm} = a_{εΒ}$

$t=0$ $\omega=0$ $a_K=0$

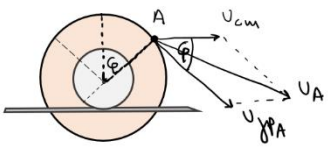


$\vec{a}_A = \vec{a}_{cm} + \vec{a}_{εΑ} \Rightarrow a_A = a_{cm} + 2a_{cm} = 3a_{cm}$

$\vec{a}_B = \vec{a}_{cm} + \vec{a}_{εΒ} \Rightarrow a_B = a_{cm} - 2a_{cm} = -a_{cm} \rightarrow a_B = a_{cm}$ μέτρο

$\frac{a_A}{a_B} = \frac{3a_{cm}}{a_{cm}} \Rightarrow \frac{a_A}{a_B} = \frac{3}{1}$

ΙΙ-γ $v_{cm} = R\omega$, $v_{γρ_A} = 2R\omega = 2v_{cm}$ $\varphi = 60^\circ \rightarrow \sigma\upsilon\nu 60^\circ = \frac{1}{2}$



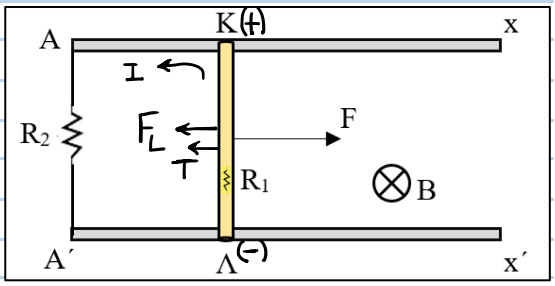
$\vec{v}_A = \vec{v}_{cm} + \vec{v}_{γρ_A} \Rightarrow v_A = \sqrt{v_{cm}^2 + v_{γρ}^2 + 2v_{cm}v_{γρ}\sigma\upsilon\nu 60^\circ}$

$v_A = \sqrt{v_{cm}^2 + 4v_{cm}^2 + 2v_{cm}\cdot 2v_{cm}\frac{1}{2}} \Rightarrow \boxed{v_A = \sqrt{7}v_{cm}}$

ΘΕΜΑ Γ

$l = 1\,m$ $R_1 = 2\,\Omega$ $m = 0{,}5\,kg$ $R_2 = 3\,\Omega$

$B = 1\,T$, $a = 2\,m/s^2$ $T = 2\,N$



$$\mathcal{E}_{επ} = \frac{\Delta\Phi}{\Delta t} = \frac{B\cdot\Delta S}{\Delta t} = \frac{B\cdot\Delta x\cdot l}{\Delta t} = Bvl$$

$v = at \rightarrow \mathcal{E}_{επ} = Bal\cdot t$

$$I = \frac{\mathcal{E}_{επ}}{R_{ολ}} = \frac{Bal}{R_{ολ}}\cdot t \Rightarrow I = \frac{2}{5}t \Rightarrow I = 0{,}4t\ SI$$

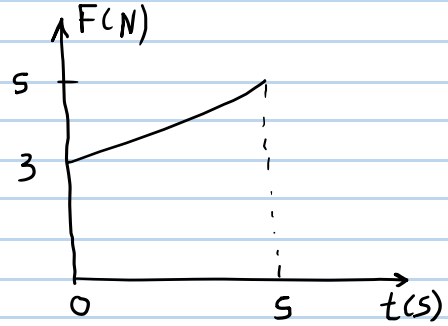
Γ1 $t = 3\,sec$ $I = 0{,}4\cdot 3 = 1{,}2A$ $V_{R_2} = IR_2 = 1{,}2\cdot 3 \Rightarrow \boxed{V_{R_2} = 3{,}6\,V}$

Γ2 $F_L = BIl = 0{,}4t\ SI$

$\Sigma F = ma \Rightarrow F - T - F_L = m\cdot a$

$\Rightarrow F - 2 - 0{,}4t = 1$

$\Rightarrow \boxed{F = 3 + 0{,}4t}\ SI$



Γ3 Η ενέργεια που προσφέρεται μέσω του έργου της F μετατρέπεται σε κινητική ενέργεια του αμαξιού Κ1, σε θερμότητα λόγω τριβής και θερμότητα λόγω φαινομένου Joule.

ΑΔΕ: $W_F = K_{τελ} + Q_{τριβης} + Q_{R_{ολ}}$

όπου $K_{τελ} = \frac{1}{2}mv^2$, $v = at = 10\,m/s \rightarrow K_{τελ} = \frac{1}{2}0{,}5\cdot 100 = 25\,J$

$Q_{τριβης} = |W_T| = |-T\cdot x|$, $x = \frac{1}{2}at^2 = 25m \rightarrow Q_{τριβης} = 50\,J$

$\Rightarrow 108{,}3 = 25 + 50 + Q_{R_{ολ}} \Rightarrow Q_{R_{ολ}} = 33{,}3\,J$

Γ4 $\Delta K = W_{\Sigma F} \rightarrow \frac{dK}{dt} = \frac{dW_{\Sigma F}}{dt} = \Sigma F\cdot\frac{dx}{dt} = \Sigma F\cdot v = m\cdot a\cdot v$

Τιν $t = 2\,sec$ $v = at = 4\,m/s \rightarrow \frac{dK}{dt} = 0{,}5\cdot 2\cdot 4 \Rightarrow \boxed{\frac{dK}{dt} = 4\,J/s}$

Γ5 Τιν $t = 5\,sec$ $F = max = 5N$. και $v = at = 10\,m/s$.

Η $\Sigma F = F - T - F_L$ μηδενίζεται οποτε οταν $\Sigma F = 0 \Rightarrow v = max = v_{op}$

$\Sigma F = 0 \Rightarrow F - T = F_L \Rightarrow F - T = B\frac{Bv_{op}l}{R_{ολ}}l \Rightarrow F - T = \frac{B^2l^2}{R_{ολ}}v_{op} \Rightarrow v_{op} = 15\,m/s.$

ΑΔΕ από τη στιγμή που $υ = 10\,m/s$ έως $υ = υ_{op} = 15\,m/s$

$$W_F + K_{αρχ} = |Q_T| + K_{τελ} + Q_{Rολ}$$

$$Fd + \frac{1}{2}mυ^2 = |-T\cdot d| + \frac{1}{2}m υ_{op}^2 + Q_{Rολ}$$

$$5d + \frac{1}{2}0{,}5\cdot 100 = 2d + \frac{1}{2}0{,}5\,225 + 6{,}25$$

$$3d = \frac{1}{4}\cdot 125 + 6{,}25 \Rightarrow 3d = 37{,}5 \Rightarrow d = 12{,}5\,m.$$

Ισχύει $\Delta q = N\frac{\Delta\Phi}{R_{ολ}} = 1\frac{B\cdot\Delta S}{R_1+R_2} = \frac{B(x+d)\cdot\ell}{R_1+R_2}$

$$\Delta q = \frac{1\cdot(25+12{,}5)\cdot 1}{5} \Rightarrow \boxed{\Delta q = 7{,}5\,C}$$

ΘΕΜΑ Δ

Δ1 Για το σώμα:

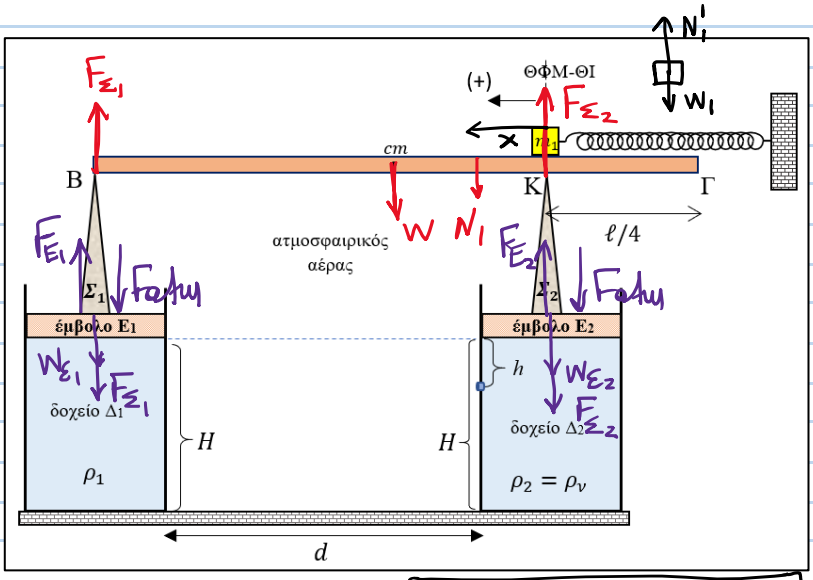
$$\Sigma F_{1y} = 0 \Rightarrow N_1' = W_1 = m_1 g = 30N$$

Για τη δοκό:

$$\Sigma F_y = 0 \Rightarrow F_{Σ_1} + F_{Σ_2} = W + N_1 \quad (1)$$

όπου $N_1 = N_1' = 30N$

$$\Sigma\tau_K = 0 \Rightarrow -\tau_{F_{Σ_1}} + \tau_W + \tau_{N_1} = 0$$



$$\Rightarrow F_{Σ_1}\frac{3\ell}{4} = W\frac{\ell}{4} + N_1\cdot x \Rightarrow 1{,}5F_{Σ_1} = 30\frac{1}{2} + 30x \Rightarrow \boxed{F_{Σ_1} = 10 + 20x \ \ SI}$$

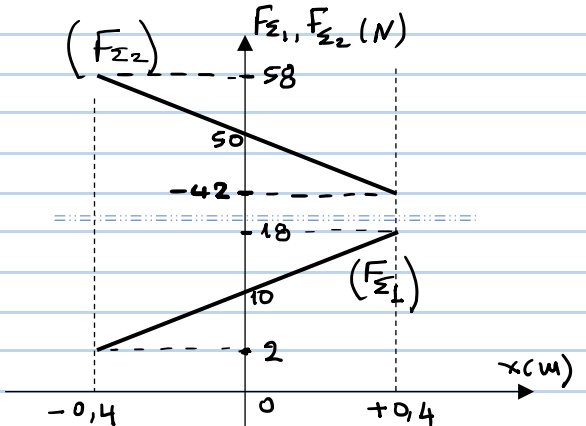
$$(1) \Rightarrow 10 + 20x + F_{Σ_2} = 30 + 30 \Rightarrow \boxed{F_{Σ_2} = 50 - 20x \ \ SI}$$

Για m_1 απλή μτ πλάτος $A = 0{,}4\,m$ οπότε $-A \le x \le +A \rightarrow -0{,}4\,m \le x \le +0{,}4\,m$

για $x = 0$ $F_{Σ_1} = 10N$, $F_{Σ_2} = 50N$

για $x = -0{,}4\,m$ $F_{Σ_1} = 2N$ $F_{Σ_2} = 58N$

για $x = +0{,}4\,m$ $F_{Σ_1} = 18N$ $F_{Σ_2} = 42N$



Δ2 Πιέσεις σε κάθε έμβολο όταν $x = +0{,}4\,m$.

Για έμβολο E_1: $\Sigma F_{E_1} = 0 \Rightarrow F_{E_1} = F_{αντλ} + W_{Σ_1} + F_{Σ_1}$

$\Rightarrow P_{E_1} S = P_{atm} S + W_{\varepsilon_1} + F_{\varepsilon_1} \Rightarrow P_{E_1} = P_{atm} + \frac{W_{\varepsilon_1} + F_{\varepsilon_1}}{S}$

$\Rightarrow P_{E_1} = 10^5 + \frac{142+18}{4\cdot 10^{-2}} = 10^5 + \frac{160}{4\cdot 10^{-2}} = 100\cdot 10^3 + 4\cdot 10^3 \Rightarrow P_{E_1} = 104\cdot 10^3 \ N/m^2$

ομοίως για εμβολο E_2: $\Sigma F_{E_2} = 0 \Rightarrow F_{E_2} = F_{atm} + W_{\varepsilon_2} + F_{\varepsilon_2}$

$\Rightarrow P_{E_2} S = P_{atm} S + W_{\varepsilon_2} + F_{\varepsilon_2} \Rightarrow P_{E_2} = P_{atm} + \frac{W_{\varepsilon_2} + F_{\varepsilon_2}}{S}$

$\Rightarrow P_{E_2} = 10^5 + \frac{38+42}{4\cdot 10^{-2}} = 10^5 + \frac{80}{4\cdot 10^{-2}} = 100\cdot 10^3 + 2\cdot 10^3 \Rightarrow P_{E_2} = 102\cdot 10^3 N/m^2$

$P_{\Pi\Theta_1} = P_{\Pi\Theta_2} \Rightarrow P_{E_1} + \rho_1 g H = P_{E_2} + \rho_v g H$

$\Rightarrow 104\cdot 10^3 + 10\cdot\rho_1 = 102\cdot 10^3 + 10^4 \Rightarrow 10\rho_1 = -2\cdot 10^3 + 10\cdot 10^3$

$\Rightarrow 10\rho_1 = 8\cdot 10^3 \Rightarrow \boxed{\rho_1 = 800 \ kg/m^3}$

Δ3 ΑΔΟ: $\vec{P}_{\pi\rho\iota\nu} = \vec{P}_{\mu\epsilon\tau\alpha} \Rightarrow \vec{P}_1 + \vec{P}_2 = \vec{P}_K \Rightarrow 0 + m_2 U_2 = (m_1+m_2) U_K$

$\Rightarrow U_K = \frac{m_2 U_2}{m_1+m_2} = \frac{2\cdot 5}{3+2} \Rightarrow U_K = 2 \ m/s$

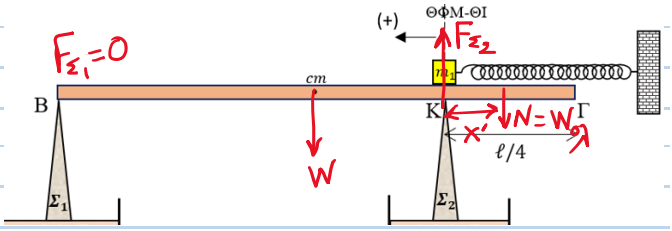
ΑΔΕΤ αμέσως μετά την κρούση: $E = K + U \Rightarrow \frac{1}{2} k A'^2 = \frac{1}{2} m_2 U_K^2 + \frac{1}{2} k x^2$

$A'^2 = \frac{m_2}{k} U_K^2 + x^2 = \frac{5}{100}\cdot 4 + \frac{16}{100} = \frac{36}{100} \Rightarrow A' = 0{,}6 \ m$

Στην οριακή περίπτωση ανατροπής

$F_{\varepsilon_1} = 0 \rightarrow \Sigma F_{\varepsilon_1} = 0$

$\Sigma \tau_K' = 0 \Rightarrow \tau_W - \tau_N = 0$



$W \frac{\ell}{4} = N|x'|$ όπου $N = W_2 = m_2 g = 50N$

$30 \frac{2}{4} = 50|x'| \Rightarrow |x'| = 0{,}3m$ από το σχήμα $\rightsquigarrow |x'| = 0{,}3m < \frac{\ell}{4} = 0{,}5m$

ανατρέπεται πριν φτάσει στο άκρο Γ σε απόσταση $x' = -0{,}3m$

Δ4 ΑΔΕΤ τη στιγμή της ανατροπής $E = K + U \Rightarrow \frac{1}{2} k A'^2 = \frac{1}{2} m_2 U^2 + \frac{1}{2} k x^2$

$U^2 = \frac{k}{m_2}(A'^2 - x'^2) = \frac{100}{5}\left(\frac{36}{100} - \frac{9}{100}\right) = \frac{100}{5}\frac{27}{100} = \frac{100}{5}\frac{27}{100} = \frac{27}{5}$

$U = \pm\sqrt{27/5} = \pm\sqrt{5{,}4} \xrightarrow{U<0} \boxed{U = -\sqrt{5{,}4} \ m/s}$

Δ5 όταν $F_{\Sigma_1} = 0 \rightarrow \Sigma F_{y_{\text{δοκού}}} = 0 \Rightarrow F'_{\Sigma_2} = W + N = 30 + 50 = 80N$

$$P'_{E_2} = P_{atm} + \frac{W_{\Sigma_2} + F'_{\Sigma_2}}{S}$$

Bernoulli: $P'_{E_2} + 0 + \rho_v g h = P_{atm} + \frac{1}{2}\rho_v v_1^2$

$$P_{atm} + \frac{W_{\Sigma_2} + F'_{\Sigma_2}}{S} + \rho_v g h = P_{atm} + \frac{1}{2}\rho_v v_1^2$$

$$v_1^2 = 2gh + \frac{2(W_{\Sigma_2} + F'_{\Sigma_2})}{\rho_v \cdot S} = 2 \cdot 10 \cdot 0,505 + \frac{2(38+80)}{10^3 \cdot 4 \cdot 10^{-2}}$$

$$v_1^2 = 10,1 + 5,9 \Rightarrow v_1^2 = 16 \Rightarrow v_1 = 4 m/s.$$

Ισχύει $\left.\begin{array}{l} x = v_1 t \Rightarrow t = x/v_1 \\ y = \frac{1}{2} g t^2 \end{array}\right\} y = \frac{g x^2}{2 v_1^2}$ όπου $x = d = 1,2 m$

$$\Rightarrow y = \frac{10 \cdot 1,44}{2 \cdot 16} \Rightarrow y = 0,45 m$$ από το έμβολο

Άρα από τη βάση το ύψος είναι:

$$h' = H - y - h = 100 cm - 45 cm - 50,5 cm$$

$$h' = 100 - 95,5 \Rightarrow \boxed{h' = 4,5 cm}$$